POLARIZATION OBSERVABLES IN D(E,E'P) AT GEV ENERGIES

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Supported by NSF grant PHY-0653312 & DOE

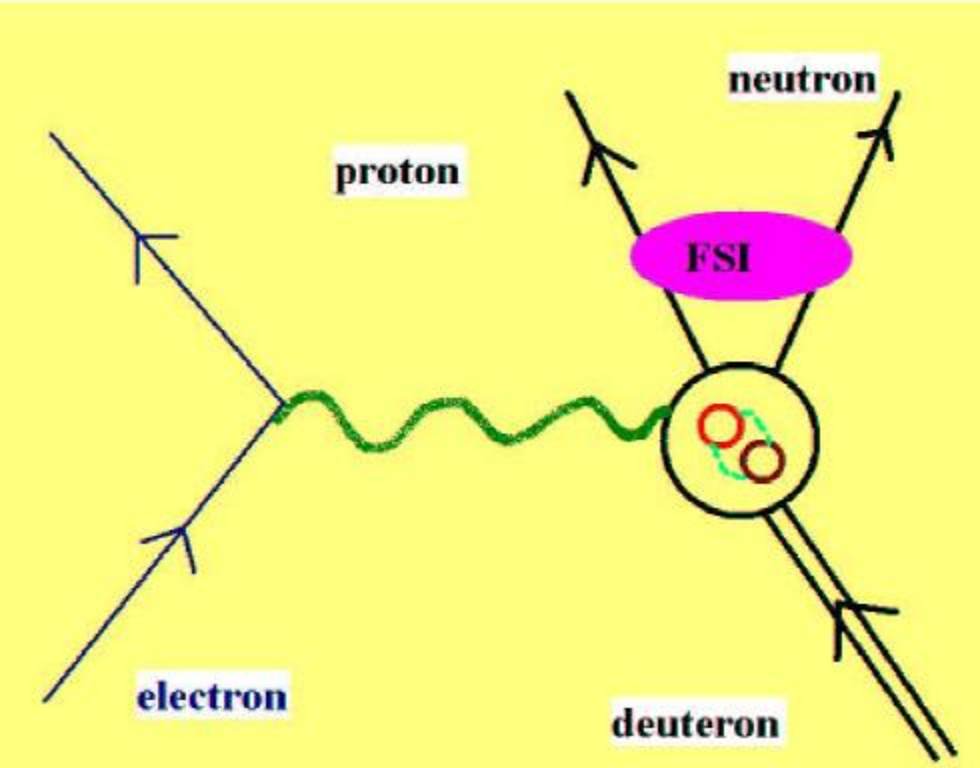
Outline

- relevance of D(e,e'p) reactions, basic observables
- a new calculation: results for unpolarized targets

  SJ & Van Orden, PRC 78, 014007 (2008)
- results for polarized targets SJ & Van Orden, PRC 80 054001 (2009)
- results for polarized ejectile protons SJ & Van Orden, PRC 81 014008 (2010)

Why D(e,e'p) reactions are interesting

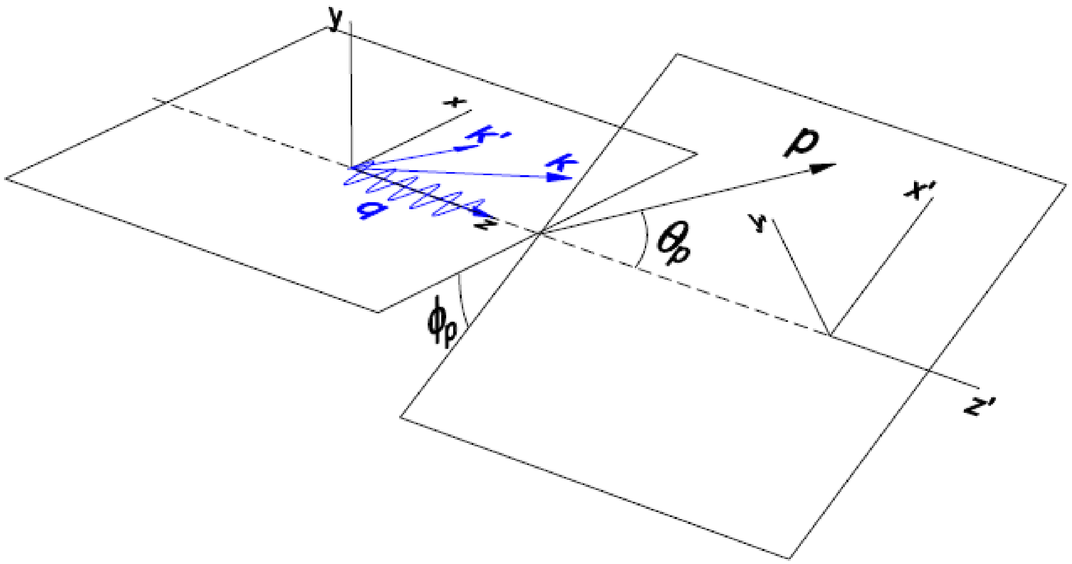
- Learn about the initial nuclear state
  - D-wave content, six-quark admixtures?
  - High momentum components
- Understand the reaction mechanism
  - Final state interaction
  - Two-body currents
  - Role of isobars
- Use the nucleus as a lab
  - Neutron form factor
  - Color transparency
- **Lots of recently published and forthcoming data!**



Differential Cross Section:

$$\left(\frac{d\sigma^5}{d\epsilon' d\Omega_e d\Omega_p}\right)_h = \frac{m_p m_n p_p}{8\pi^3 M_d}\sigma_{Mott} f_{rec}^{-1}\Big[\Big(v_L R_L + v_T R_T$$

$$+ v_{TT} R_{TT}\cos 2\phi_p + v_{LT} R_{LT}\cos\phi_p\Big) + h v_{LT'} R_{LT'}\sin\phi_p\Big]$$



Hadronic Tensor:

$$w_{\lambda'_\gamma,\lambda_\gamma} = \frac{1}{3}\sum_{s_1,s_2,\lambda_d} \langle \boldsymbol{p}_1 s_1; \boldsymbol{p}_2 s_2 | \, J_{\lambda'_\gamma} \, | \boldsymbol{P}\lambda_d \rangle^* \, \langle \boldsymbol{p}_1 s_1; \boldsymbol{p}_2 s_2 | \, J_{\lambda_\gamma} \, | \boldsymbol{P}\lambda_d \rangle$$

with $\quad J_{\pm 1} = \mp \frac{1}{\sqrt{2}}(J^1 \pm i J^2)$

is used to define the Response Functions:

$$\begin{aligned}
R_L &\equiv w_{00} \\
R_T &\equiv w_{1,1} + w_{-1,-1} \\
R_{TT}\cos 2\phi_p &\equiv 2\Re(w_{1,-1}) \\
R_{LT}\cos \phi_p &\equiv -2\Re(w_{01} - w_{0-1}) \\
R_{LT'}\sin \phi_p &\equiv -2\Re(w_{01} + w_{0-1})
\end{aligned}$$

More Observables: Asymmetries

$$A_{TT} = \frac{v_{TT}R_{TT}}{v_L R_L + v_T R_T}$$

$$A_{LT} = \frac{\sigma_0(0^\circ) - \sigma_0(180^\circ)}{\sigma_0(0^\circ) + \sigma_0(180^\circ)} = \frac{v_{LT}R_{LT}}{v_L R_L + v_T R_T + v_{TT}R_{TT}}$$

$$A_{LT'} = \frac{\sigma_{+1}(90^\circ) - \sigma_{-1}(90^\circ)}{\sigma_{+1}(90^\circ) + \sigma_{-1}(90^\circ)} = \frac{v_{LT'}R_{LT'}}{v_L R_L + v_T R_T - v_{TT}R_{TT}}$$

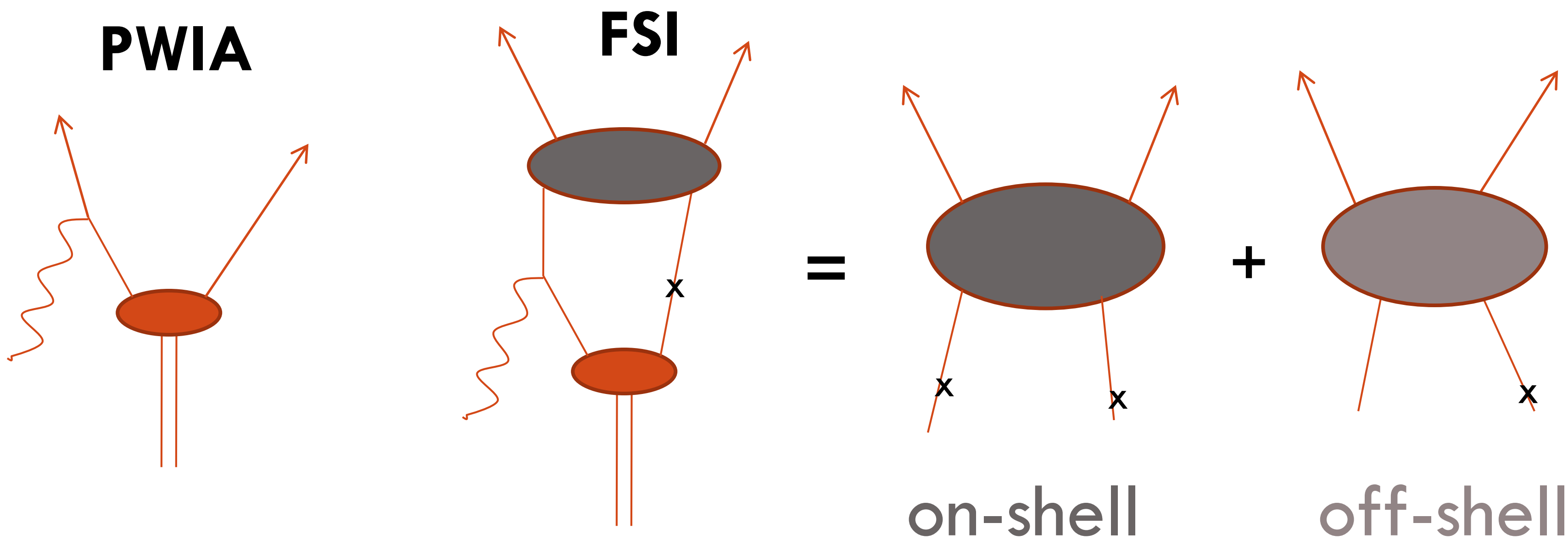
with the short hand:

$$\sigma_h(\phi_p) \equiv \left(\frac{d\sigma^5}{d\epsilon' d\Omega_e d\Omega_p}\right)_h$$

A New Calculation

- Relativistic deuteron w. f.: solution of Gross eqn.
- one-body e.m. current $\Gamma^{\mu}(q) = F_1(Q^2)\gamma^{\mu} + \frac{F_2(Q^2)}{2m} i\sigma^{\mu\nu} q_{\nu}$
- Calculation without forward angle approximations
- Up-to-date SAID parameterization of the NN scattering amplitude used
- All parts of the NN amplitude included
  - Central
  - Spin-orbit
  - Double spin-flip

Latest SAID analysis for pn scattering up to 1.3GeV, see Arndt, Briscoe, Strakovsky, Workman, Phys.Rev.C76:025209,2007



- Saclay amplitudes: $M(\vec{k}', \vec{k}) = \frac{1}{2} \; [(a+b) + (a-b)\sigma_{1,n}\sigma_{2,n}$
  $+ \; (c+d)\sigma_{1,m}\sigma_{2,m} + (c-d)\sigma_{1,l}\sigma_{2,l} + e(\sigma_{1,n} + \sigma_{2,n})]$

- Invariant amplitudes (McNeil, Ray, Wallace)

$$F = F_S + F_V\,\gamma_1{\cdot}\gamma_2 + F_T\,\sigma_1^{\mu\nu}\sigma_{2,\mu\nu} + F_P\,\gamma_1^5\gamma_2^5 + F_A\,\gamma_1^5\gamma_1^\mu\gamma_2^5\gamma_{2\mu}$$

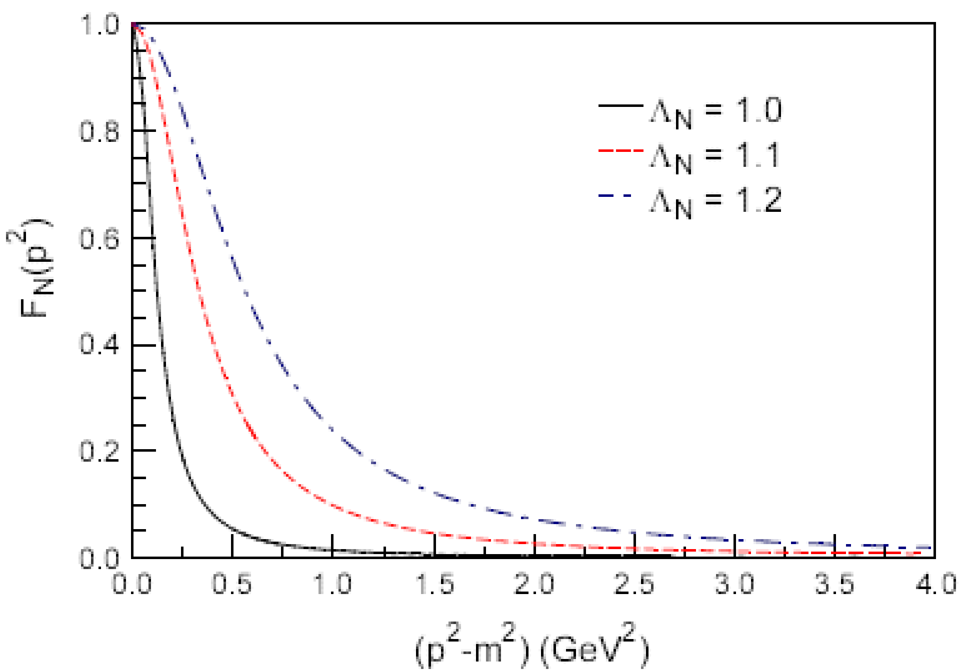
Positive Energy off-shell FSI prescription:

-retain the five on-shell invariants

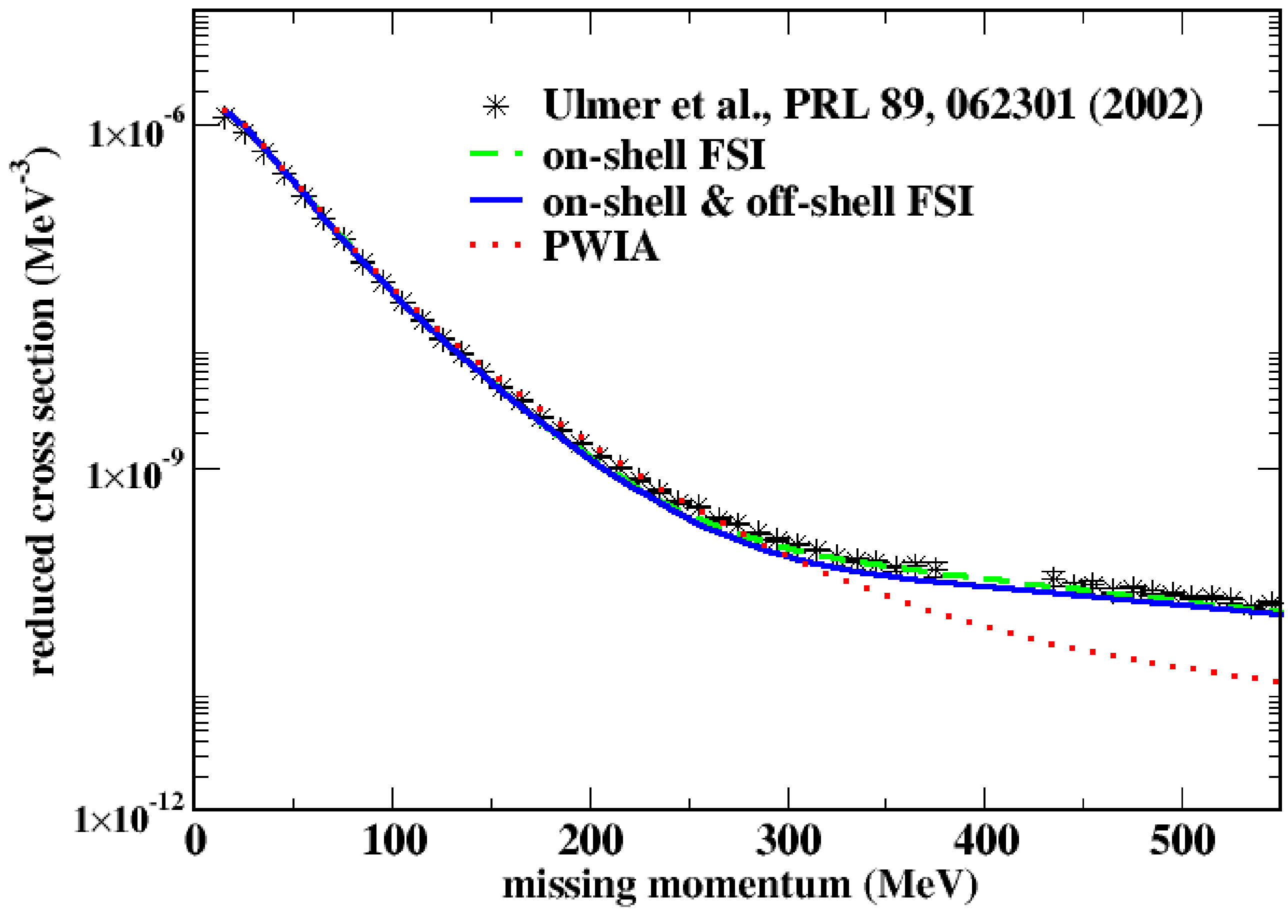
$$\mathcal{F}_i(s,t) \rightarrow \mathcal{F}_i(s,t,u) F_N(s+t+u-3m^2)$$

- use a form factor

$$F_N(p^2) = \frac{(\Lambda_N^2 - m^2)^2}{(p^2-m^2)^2 + (\Lambda_N^2 - m^2)^2}$$

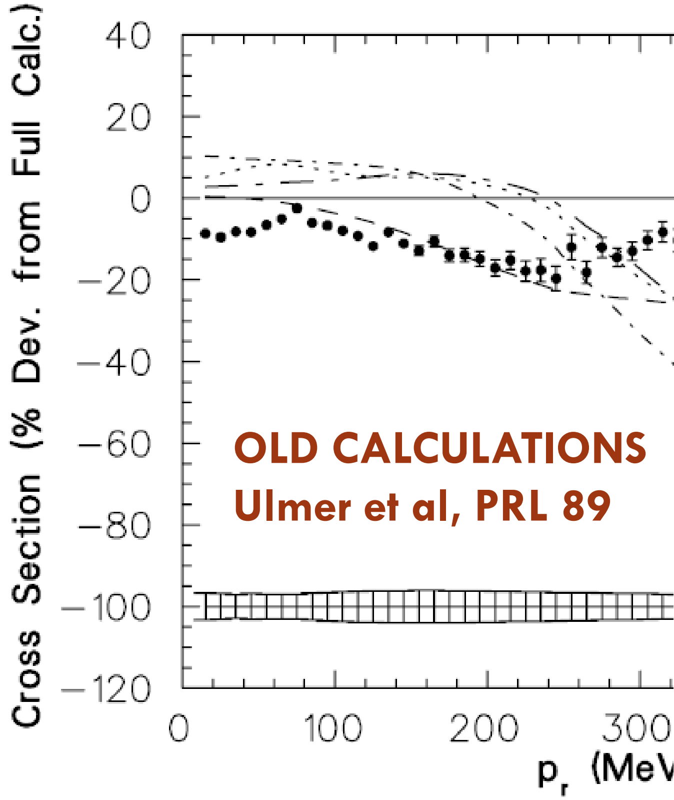


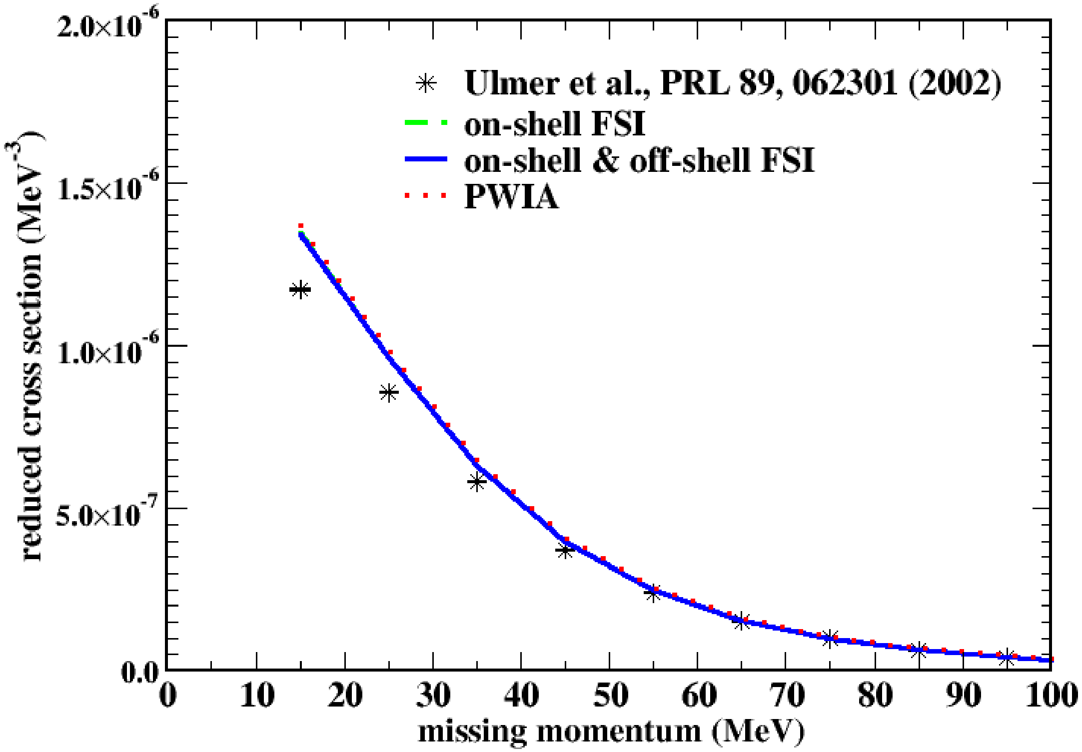
Diff. Cross Section Data from Hall A



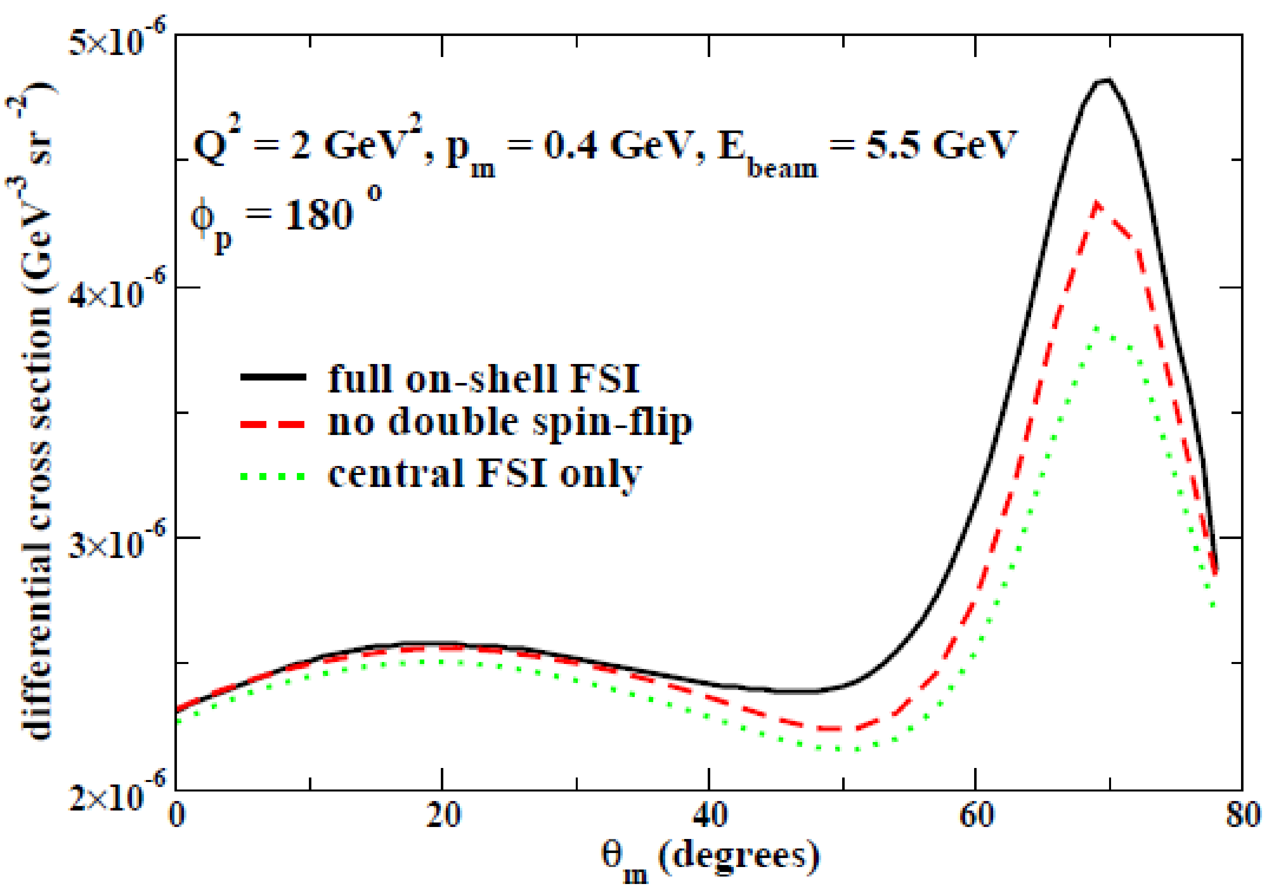
data: Ulmer et al,
PRL 89 (2002)

theory: SJ & JW Van Orden,
Phys.Rev.C78:014007, 2008

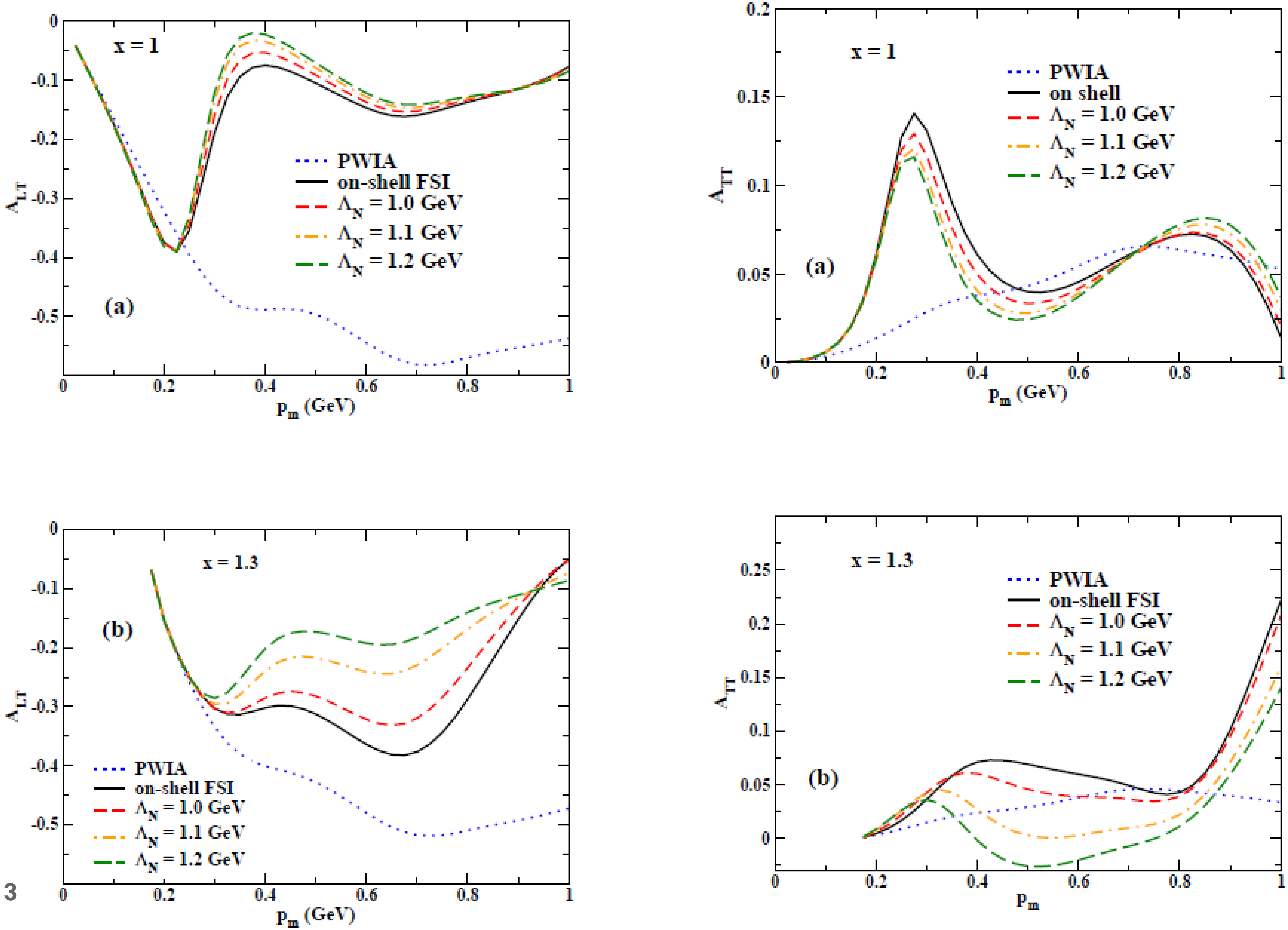

OLD CALCULATIONS
Ulmer et al, PRL 89
Cross Section (% Dev. from Full Calc.)
p_r (MeV


Ulmer et al., PRL 89, 062301 (2002)
on-shell FSI
on-shell & off-shell FSI
PWIA
reduced cross section (MeV^{-3})
missing momentum (MeV)

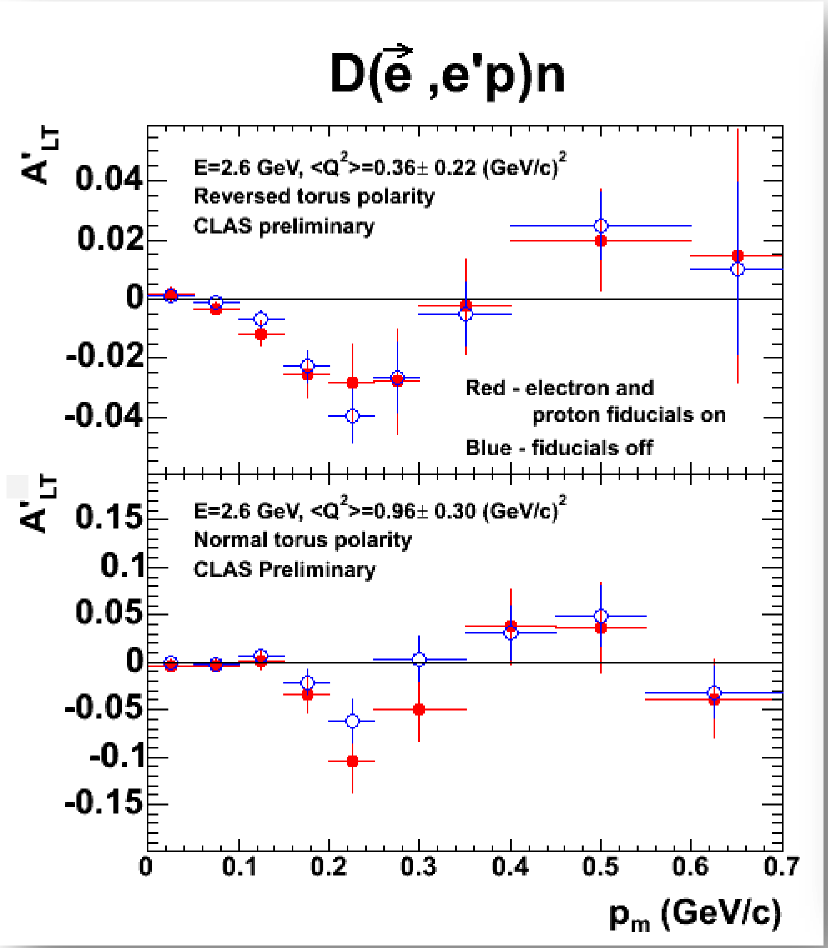
Influence of the NN amplitude

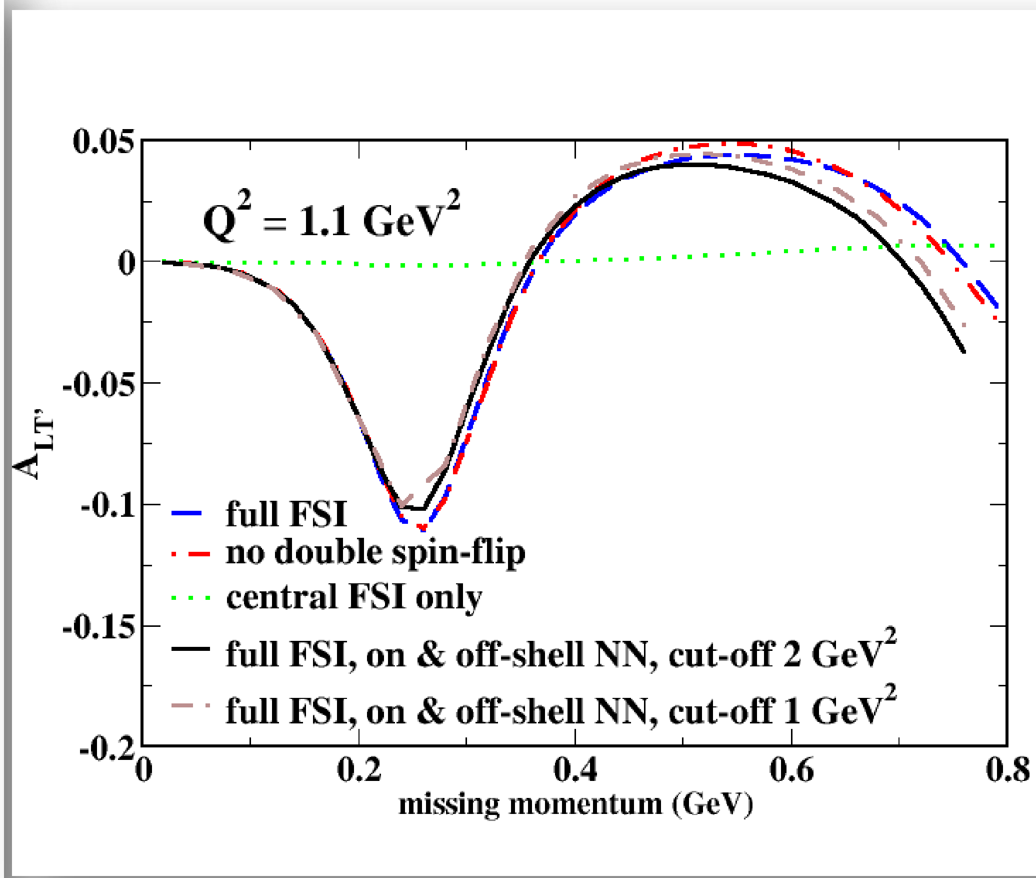


Off-shell FSI Influence, Uncertainties



LT' Asymmetry $A_{LT'} = \frac{v_{LT'} R_{LT'}}{v_L R_L + v_T R_T + v_{TT} R_{TT}}$





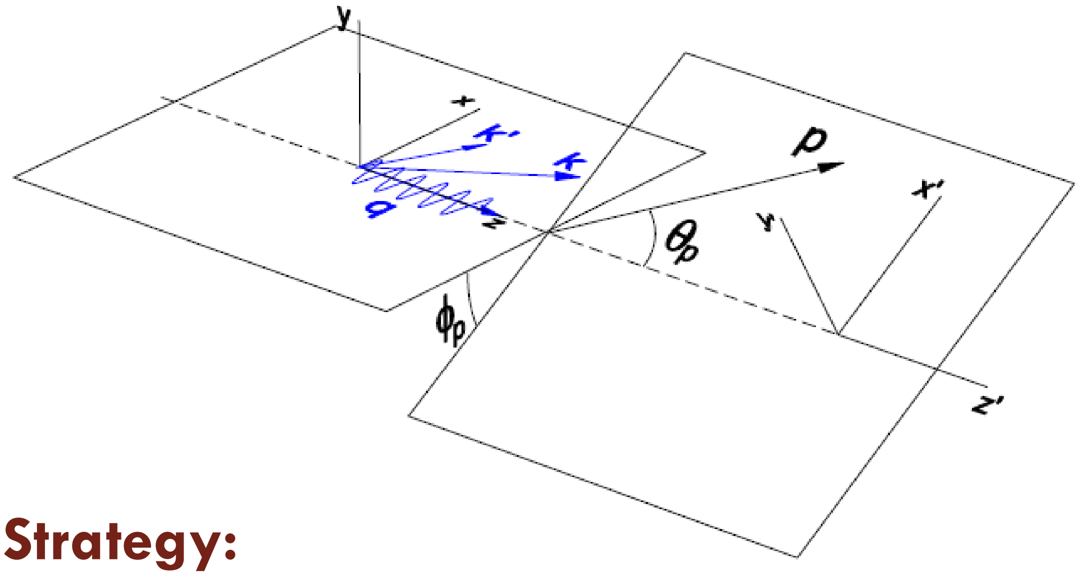
Jefferson Lab, **CLAS Preliminary Data**, Jerry Gilfoyle

Polarized Deuteron Targets

- deuteron has spin 1, M_J = -1, 0, +1
- deuteron can be **vector** polarized: $n_+ - n_-$

  or **tensor** polarized: $n_+ + n_- - 2n_0$
- **polarization axis**
  - theorist's choice: along the three-momentum transfer $\vec{q}$
  - experimentalist's choice: along the beam, along …

  SJ & Van Orden, PRC 80 054001 (2009)



Strategy:

1) define reduced responses in the **hadron plane**, this makes any Φ_p dependence **explicit**

2) use a **density matrix** to handle any type of deuteron polarization, e.g. T_{10} and T_{20}

3) **rotate** the density matrix to accommodate a **polarization axis** along the beam (or any other direction)

1) Define **reduced responses** in the **hadron plane**, this makes any **Φ_p dependence explicit**:

$$
\begin{aligned}
R_L(\overline{D}) &= \overline{R}_L^{(I)}(\overline{D}) \\
R_T(\overline{D}) &= \overline{R}_T^{(I)}(\overline{D}) \\
R_{TT}(\overline{D}) &= \overline{R}_{TT}^{(I)}(\overline{D}) \cos 2\phi_p + \overline{R}_{TT}^{(II)}(\overline{D}) \sin 2\phi_p \\
R_{LT}(\overline{D}) &= \overline{R}_{LT}^{(I)}(\overline{D}) \cos \phi_p + \overline{R}_{LT}^{(II)}(\overline{D}) \sin \phi_p \\
R_{LT'}(\overline{D}) &= \overline{R}_{LT'}^{(I)}(\overline{D}) \sin \phi_p + \overline{R}_{LT'}^{(II)}(\overline{D}) \cos \phi_p \\
R_{T'}(\overline{D}) &= \overline{R}_{T'}^{(II)}(\overline{D})
\end{aligned}
$$

$$
\begin{aligned}
\overline{R}_L^{(I)}(\overline{D}) &= \sum_i \overline{R}_L^{(I)}(\boldsymbol{\tau}_i^{(I)})\overline{T}_i^{(I)} = \overline{w}_{00}(\overline{D}) \\
\overline{R}_T^{(I)}(\overline{D}) &= \sum_i \overline{R}_T^{(I)}(\boldsymbol{\tau}_i^{(I)})\overline{T}_i^{(I)} = \overline{w}_{1,1}(\overline{D}) + \overline{w}_{-1,-1}(\overline{D}) \\
\overline{R}_{TT}^{(I)}(\overline{D}) &= \sum_i \overline{R}_{TT}^{(I)}(\boldsymbol{\tau}_i^{(I)})\overline{T}_i^{(I)} = 2\Re(\overline{w}_{1,-1}(\overline{D})) \\
\overline{R}_{TT}^{(II)}(\overline{D}) &= \sum_i \overline{R}_{TT}^{(II)}(\boldsymbol{\tau}_i^{(II)})\overline{T}_i^{(II)} = 2\Im(\overline{w}_{1,-1}(\overline{D})) \\
\overline{R}_{LT}^{(I)}(\overline{D}) &= \sum_i \overline{R}_{LT}^{(I)}(\boldsymbol{\tau}_i^{(I)})\overline{T}_i^{(I)} = -2\Re(\overline{w}_{01}(\overline{D}) - \overline{w}_{0-1}(\overline{D})) \\
\overline{R}_{LT}^{(II)}(\overline{D}) &= \sum_i \overline{R}_{LT}^{(II)}(\boldsymbol{\tau}_i^{(II)})\overline{T}_i^{(II)} = 2\Im(\overline{w}_{01}(\overline{D}) + \overline{w}_{0-1}(\overline{D})) \\
\overline{R}_{LT'}^{(I)}(\overline{D}) &= \sum_i R_{LT'}^{(I)}(\boldsymbol{\tau}_i^{(I)})\overline{T}_i^{(I)} = 2\Im(\overline{w}_{01}(\overline{D}) - \overline{w}_{0-1}(\overline{D})) \\
R_{LT'}^{(II)}(\overline{D}) &= \sum_i \overline{R}_{LT'}^{(II)}(\boldsymbol{\tau}_i^{(II)})\overline{T}_i^{(II)} = -2\Re(\overline{w}_{01}(\overline{D}) + \overline{w}_{0-1}(\overline{D})) \\
\overline{R}_{T'}^{(II)}(\overline{D}) &= \sum_i \overline{R}_{T'}^{(II)}(\boldsymbol{\tau}_i^{(II)})\overline{T}_i^{(II)} = \overline{w}_{1,1}(\overline{D}) - \overline{w}_{-1,-1}(\overline{D}) \,,
\end{aligned}
$$

The interference reduced responses are either real or imaginary parts of the hadronic tensor.

$$
\begin{aligned}
\overline{T}_i^{(I)} &\in \{U, \Im(\overline{T}_{11}), \overline{T}_{20}, \Re(\overline{T}_{21}), \Re(\overline{T}_{22})\} \\
\overline{T}_i^{(II)} &\in \{\overline{T}_{10}, \Re(\overline{T}_{11}), \Im(\overline{T}_{21}), \Im(\overline{T}_{22})\}
\end{aligned}
$$

2) Use a **density matrix** to handle any type of deuteron polarization, e.g. T_{10} and T_{20}

$$\rho = \frac{1}{3}\begin{pmatrix} 1+\sqrt{\frac{3}{2}}T_{10}+\frac{1}{\sqrt{2}}T_{20} & -\sqrt{\frac{3}{2}}(T_{11}^*+T_{21}^*) & \sqrt{3}\,T_{22}^* \\ -\sqrt{\frac{3}{2}}(T_{11}+T_{21}) & 1-\sqrt{2}\,T_{20} & -\sqrt{\frac{3}{2}}(T_{11}^*-T_{21}^*) \\ \sqrt{3}\,T_{22} & -\sqrt{\frac{3}{2}}(T_{11}-T_{21}) & 1-\sqrt{\frac{3}{2}}T_{10}+\frac{1}{\sqrt{2}}T_{20} \end{pmatrix}$$

T_{ij}: tensor polarization coefficients, experimental input

$$w_{\lambda'_\gamma,\lambda_\gamma}(D) = \sum_{s_1,s_2,\lambda_d,\lambda'_d} \langle \boldsymbol{p}_1 s_1; \boldsymbol{p}_2 s_2; (-)|\, J_{\lambda'_\gamma}\, |\boldsymbol{P}\lambda'_d\rangle^* \langle \boldsymbol{p}_1 s_1; \boldsymbol{p}_2 s_2; (-)|\, J_{\lambda_\gamma}\, |\boldsymbol{P}\lambda_d\rangle\, \rho_{\lambda_d\lambda'_d}$$

hadronic tensor, with the density matrix

3) **rotate** the density matrix

$$\overline{\rho}_{\lambda_d\lambda'_d} = \sum_{\Lambda\Lambda'} D^1_{\lambda_d\Lambda}(-\phi_p,\theta_{kq},0)\, D^1_{\lambda'_d\Lambda'}(-\phi_p,\theta_{kq},0)\, \tilde{\rho}^D_{\Lambda\Lambda'}$$

Target Polarization Observables

vector asym.: $$A_d^V = \frac{v_L R_L(\widetilde{T}_{10}) + v_T R_T(\widetilde{T}_{10}) + v_{TT} R_{TT}(\widetilde{T}_{10}) + v_{LT} R_{LT}(\widetilde{T}_{10})}{\widetilde{T}_{10}\Sigma}$$

tensor asym.: $$A_d^T = \frac{v_L R_L(\widetilde{T}_{20}) + v_T R_T(\widetilde{T}_{20}) + v_{TT} R_{TT}(\widetilde{T}_{20}) + v_{LT} R_{LT}(\widetilde{T}_{20})}{\widetilde{T}_{20}\Sigma}$$

beam vector asym.: $$A_{ed}^V = \frac{v_{LT'} R_{LT'}(\widetilde{T}_{10}) + v_{T'} R_{T'}(\widetilde{T}_{10})}{\widetilde{T}_{10}\Sigma}$$

beam tensor asym.: $$A_{ed}^T = \frac{v_{LT'} R_{LT'}(\widetilde{T}_{20}) + v_{T'} R_{T'}(\widetilde{T}_{20})}{\widetilde{T}_{20}\Sigma}$$

denominator, unpolarized: $$\Sigma = v_L R_L(U) + v_T R_T(U) + v_{TT} R_{TT}(U) + v_{LT} R_{LT}(U)$$

From **parity** and **time reversal** invariance:

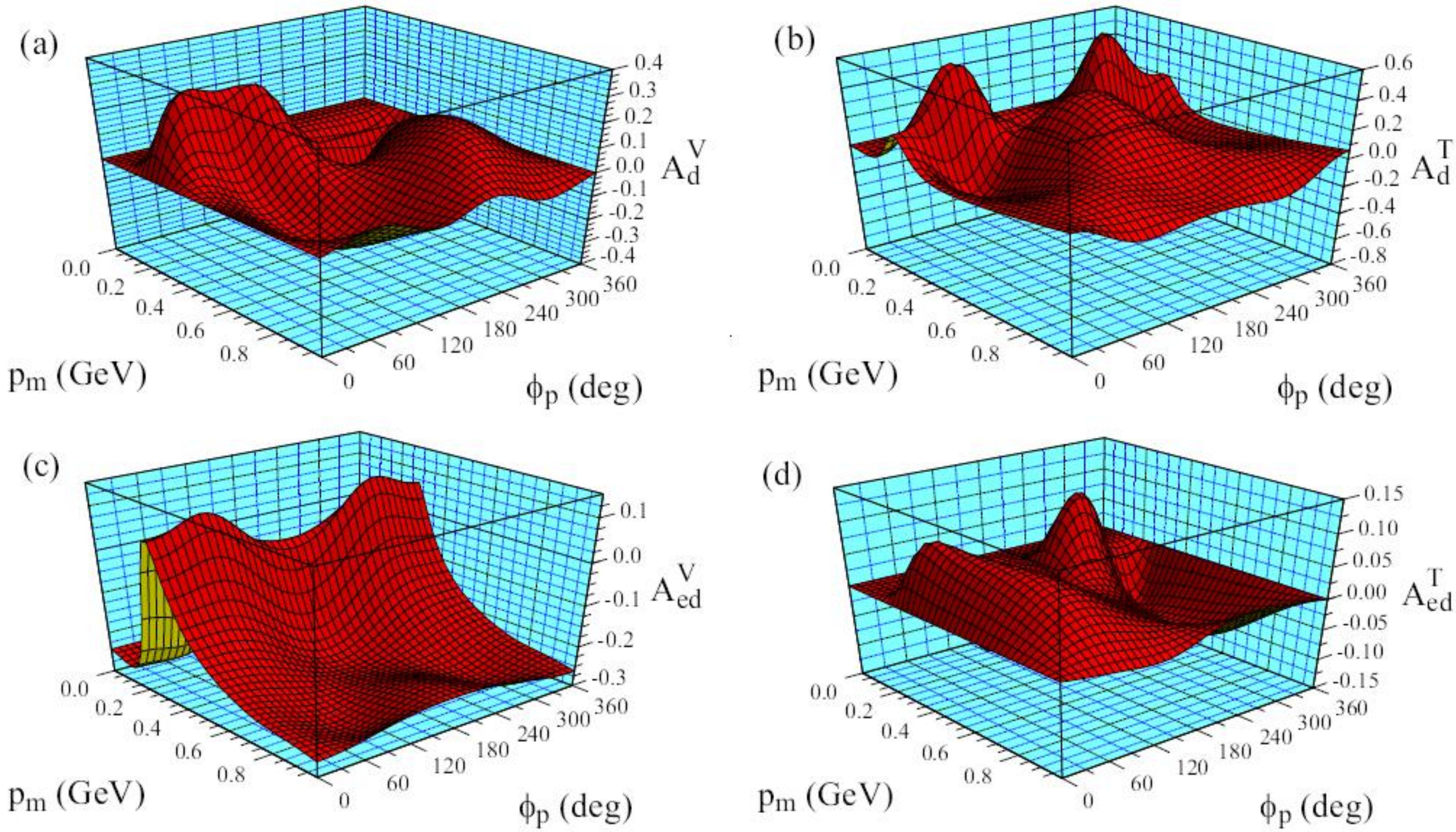
$$\overline{\langle \boldsymbol{p}_1 s_1; \boldsymbol{p}_2 s_2; (-)| \, J_{\lambda_\gamma} \, |\boldsymbol{P}\lambda_d\rangle} = \overline{\langle \boldsymbol{P}\lambda_d| \, J_{\lambda_\gamma} \, |\boldsymbol{p}_1 s_1; \boldsymbol{p}_2 s_2; (+)\rangle}$$
$$= \overline{\langle \boldsymbol{p}_1 s_1; \boldsymbol{p}_2 s_2; (+)| \, J_{\lambda_\gamma} \, |\boldsymbol{P}\lambda_d\rangle}^{*}$$

In **PWIA**: no difference between (+) and (-) boundary conditions

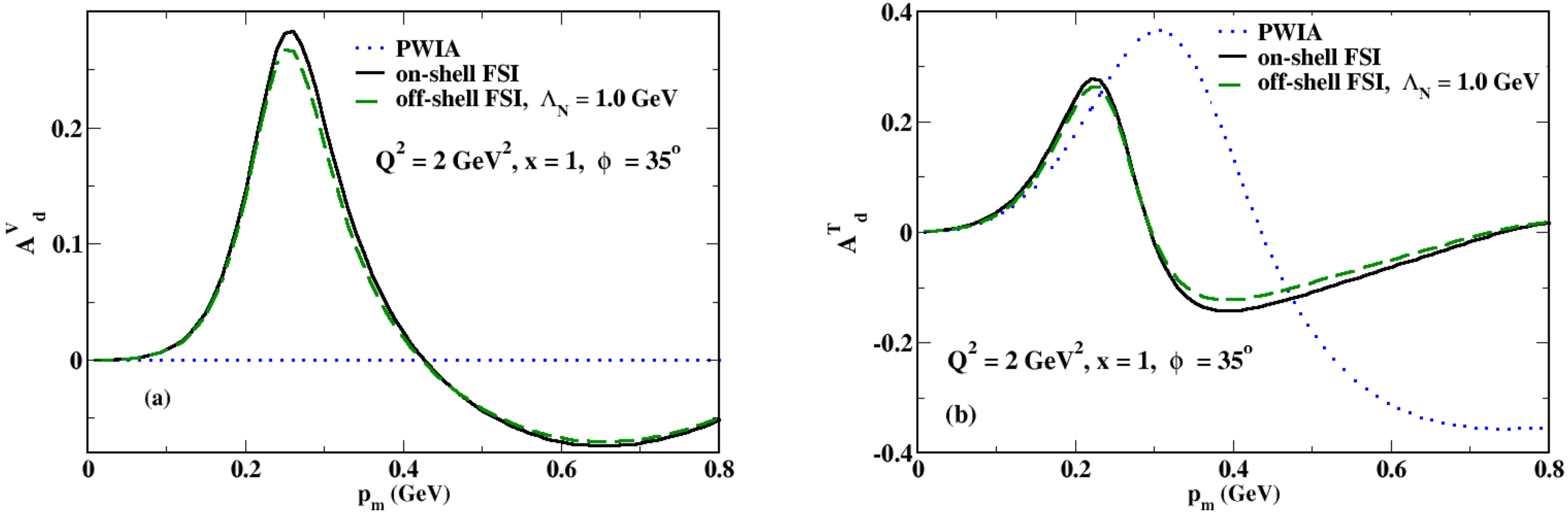
→ matrix elements must be real

→ **A^V_d and A^T_{ed} vanish in PWIA**!

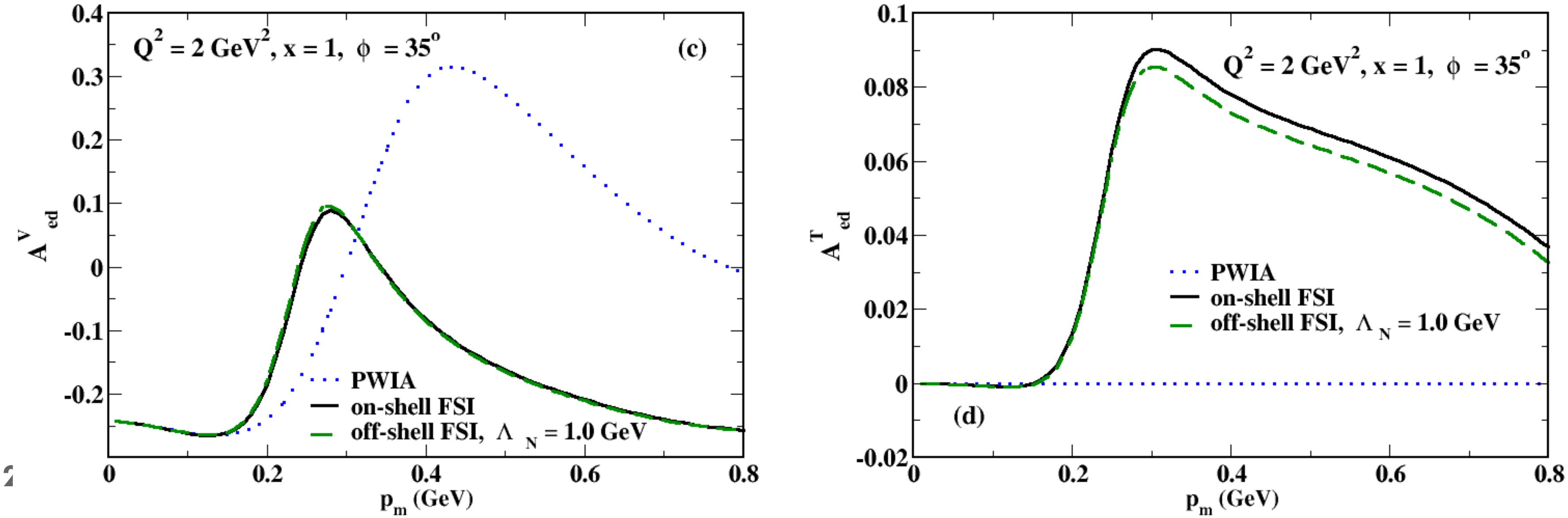
$$\overline{T}_i^{(I)} \in \{U, \Im(\overline{T}_{11}), \overline{T}_{20}, \Re(\overline{T}_{21}), \Re(\overline{T}_{22})\}$$
$$\overline{T}_i^{(II)} \in \{\overline{T}_{10}, \Re(\overline{T}_{11}), \Im(\overline{T}_{21}), \Im(\overline{T}_{22})\}$$

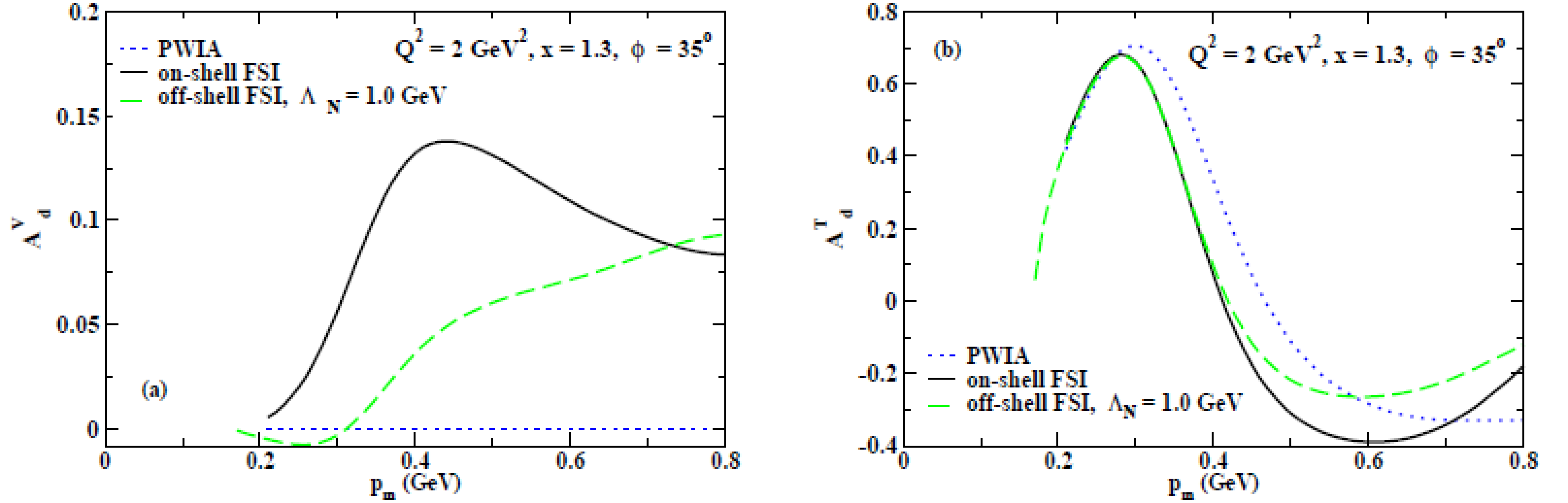


on-shell FSI, $x = 1$, $Q^2 = 2$ GeV^2

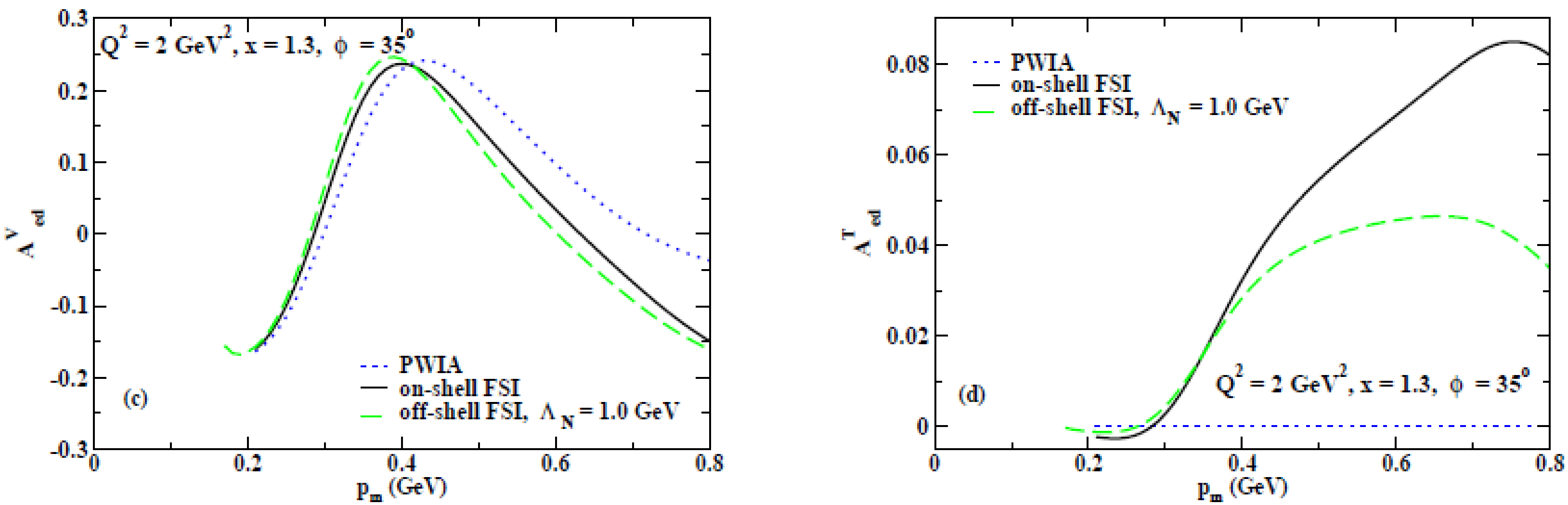


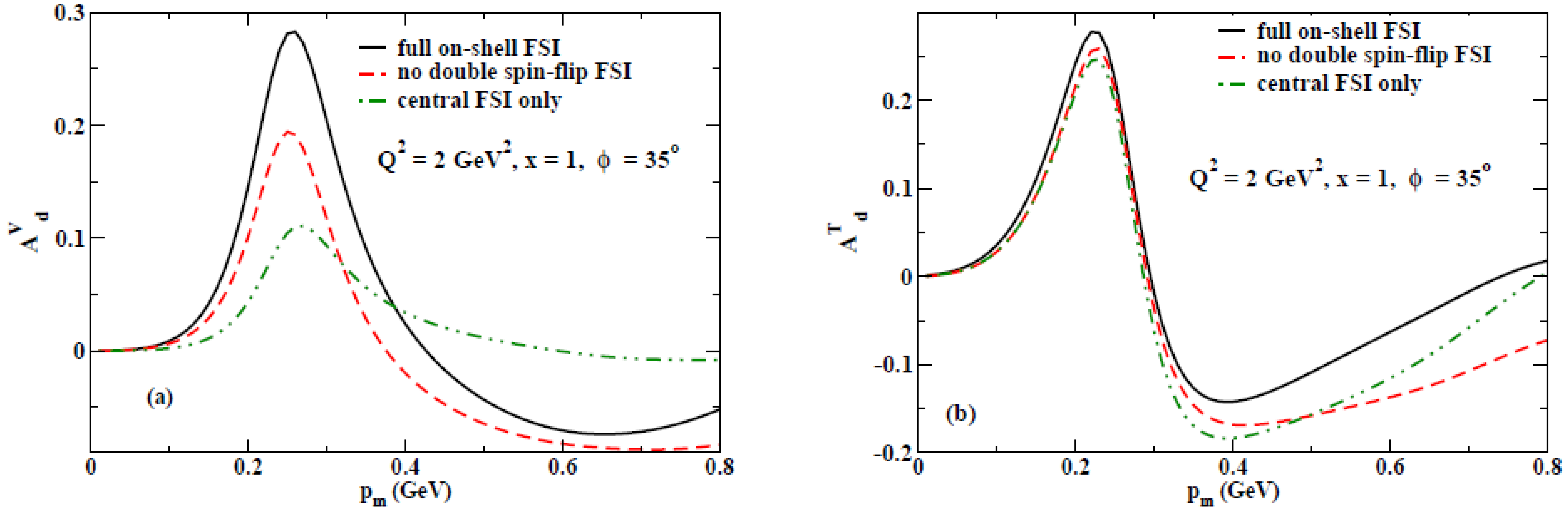
Momentum Distributions x = 1, Q^2 = 2 GeV^2



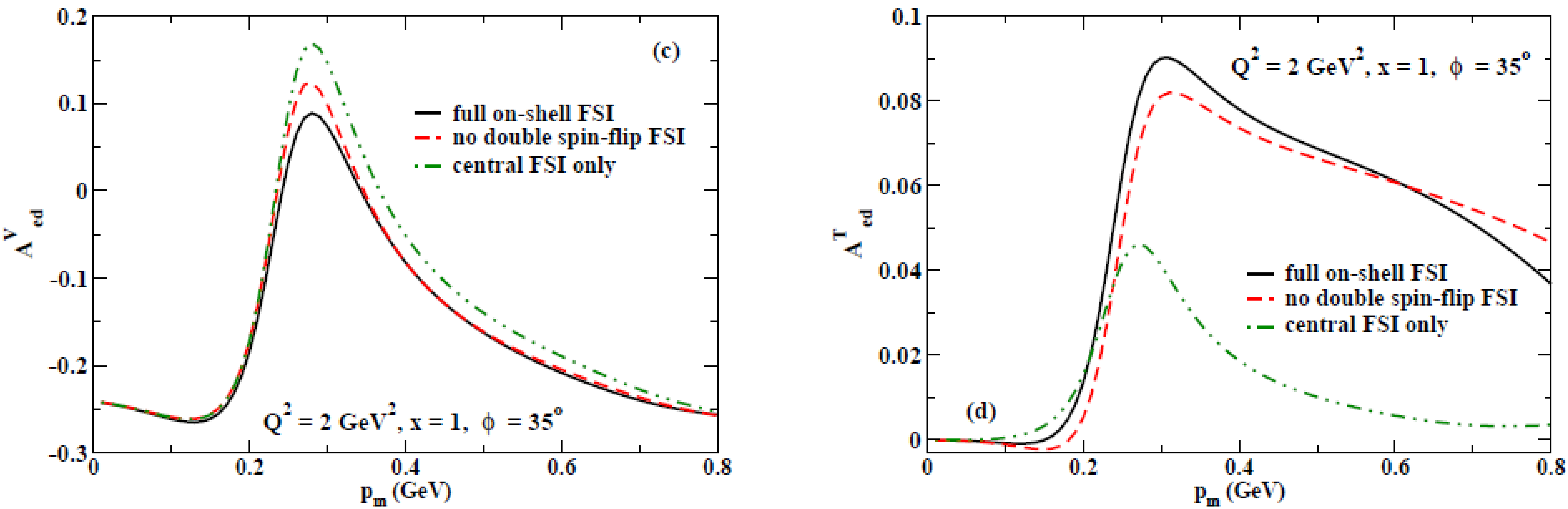


Momentum Distributions x = 1.3, $Q^2 = 2$ GeV^2





Role of Spin-Dependent FSIs: Single Spin Flip and Double Spin Flip



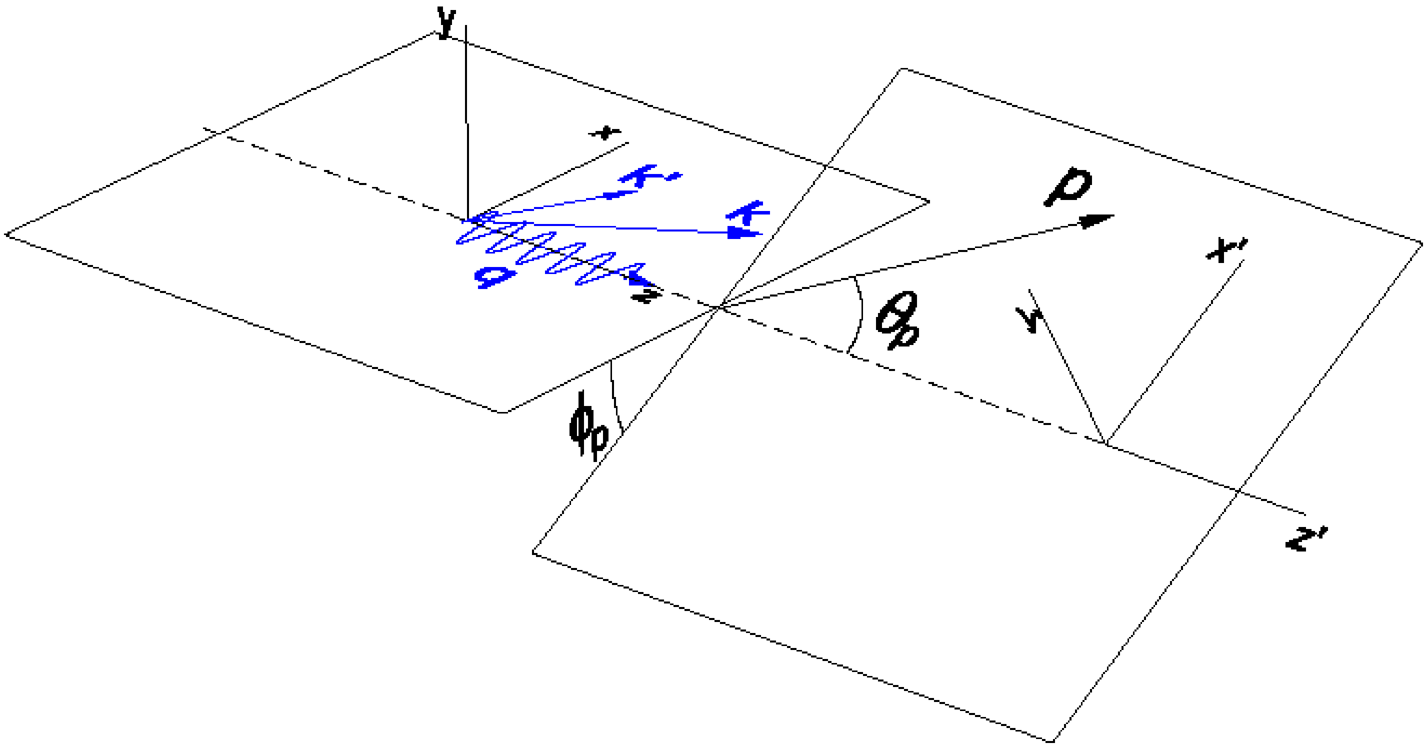
Summary

- New, relativistic calculation available
- Full, up-to-date NN scattering amplitude employed
  - Spin-dependent terms are important
  - just central FSI is insufficient even for the quasi-elastic region x = 1
- Agreement with data is encouraging
- four asymmetries have been considered, two each are similar
- FSIs and ground state information are entangled
- Wishlist: measurement of A^{V}_{d} or A^{T}_{ed} at larger x

Outlook

- ejectile polarization calculation done
- isobar contributions and meson exchange currents
- have theory, will calculate for experimentalists ☺

Polarized Ejected Proton



Polarized Ejected Proton

hadronic tensor:

$$w_{\lambda'_\gamma,\lambda_\gamma}(\hat{\mathcal{S}}) = \frac{2}{3} \sum_{s_1,s'_1,s_2,\lambda_d} \langle \boldsymbol{p}_1 s'_1; \boldsymbol{p}_2 s_2; (-)| \, J_{\lambda'_\gamma} \, |\boldsymbol{P}\lambda_d\rangle^* \, \langle \boldsymbol{p}_1 s_1; \boldsymbol{p}_2 s_2; (-)| \, J_{\lambda_\gamma} \, |\boldsymbol{P}\lambda_d\rangle \, \mathcal{P}_{s'_1 s_1}(\hat{\mathcal{S}})$$

with the **spin projection operator** $\quad \mathcal{P}(\hat{\mathcal{S}}) = \frac{1}{2}\left(\mathbf{1} + \boldsymbol{\sigma}\cdot\hat{\mathcal{S}}\right)$

define normal, longitudinal, sideways directions:

$$\begin{aligned} \hat{n} &= \hat{y}' \\ \hat{l} &= \sin\theta_p \, \hat{x}' + \cos\theta_p \, \hat{z}' \\ \hat{s} &= \cos\theta_p \, \hat{x}' - \sin\theta_p \, \hat{z}' \end{aligned}$$

$$\boldsymbol{\sigma}\cdot\hat{\overline{\mathcal{S}}} = \boldsymbol{\sigma}\cdot\hat{n}\;\hat{n}\cdot\hat{\overline{\mathcal{S}}} + \boldsymbol{\sigma}\cdot\hat{l}\;\hat{l}\cdot\hat{\overline{\mathcal{S}}} + \boldsymbol{\sigma}\cdot\hat{s}\;\hat{s}\cdot\hat{\overline{\mathcal{S}}}$$

define unpolarized, normal, longitudinal and sideways responses:

$$\overline{R}_K^{(I)}(\hat{\mathcal{S}}) = \overline{R}_K(\mathbf{1}) + \overline{R}_K(\boldsymbol{\sigma}\cdot\hat{n})\hat{n}\cdot\hat{\mathcal{S}}$$

$$\overline{R}_K^{(II)}(\hat{\mathcal{S}}) = \overline{R}_K(\boldsymbol{\sigma}\cdot\hat{l})\hat{l}\cdot\hat{\mathcal{S}} + \overline{R}_K(\boldsymbol{\sigma}\cdot\hat{s})\hat{s}\cdot\hat{\mathcal{S}}$$

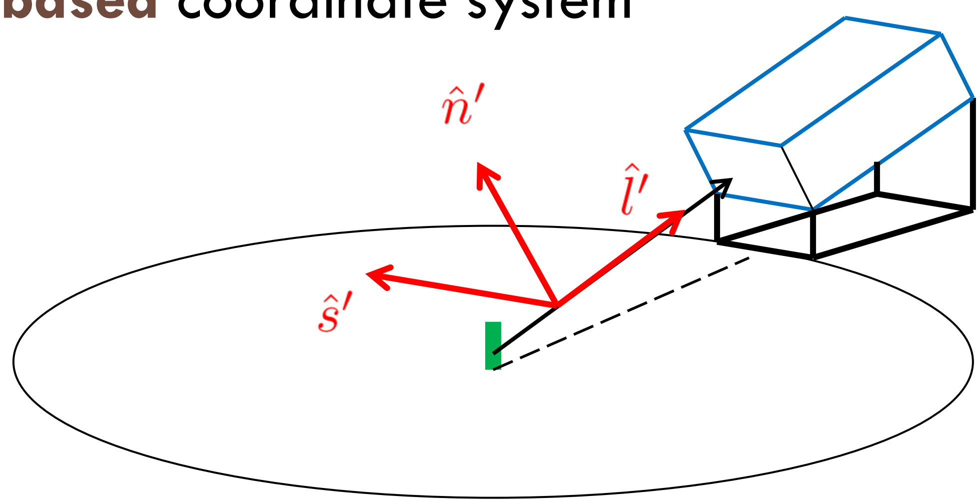
problem: hadron plane, and azimuthal angle, are ill defined for $\theta_p = 0°$

solution: use a **spectrometer based** coordinate system

$$\hat{l}' = \hat{l}$$

$$\hat{s}' = \frac{\hat{y}\times\hat{l}}{|\hat{y}\times\hat{l}|}$$

$$\hat{n}' = \hat{l}'\times\hat{s}'.$$



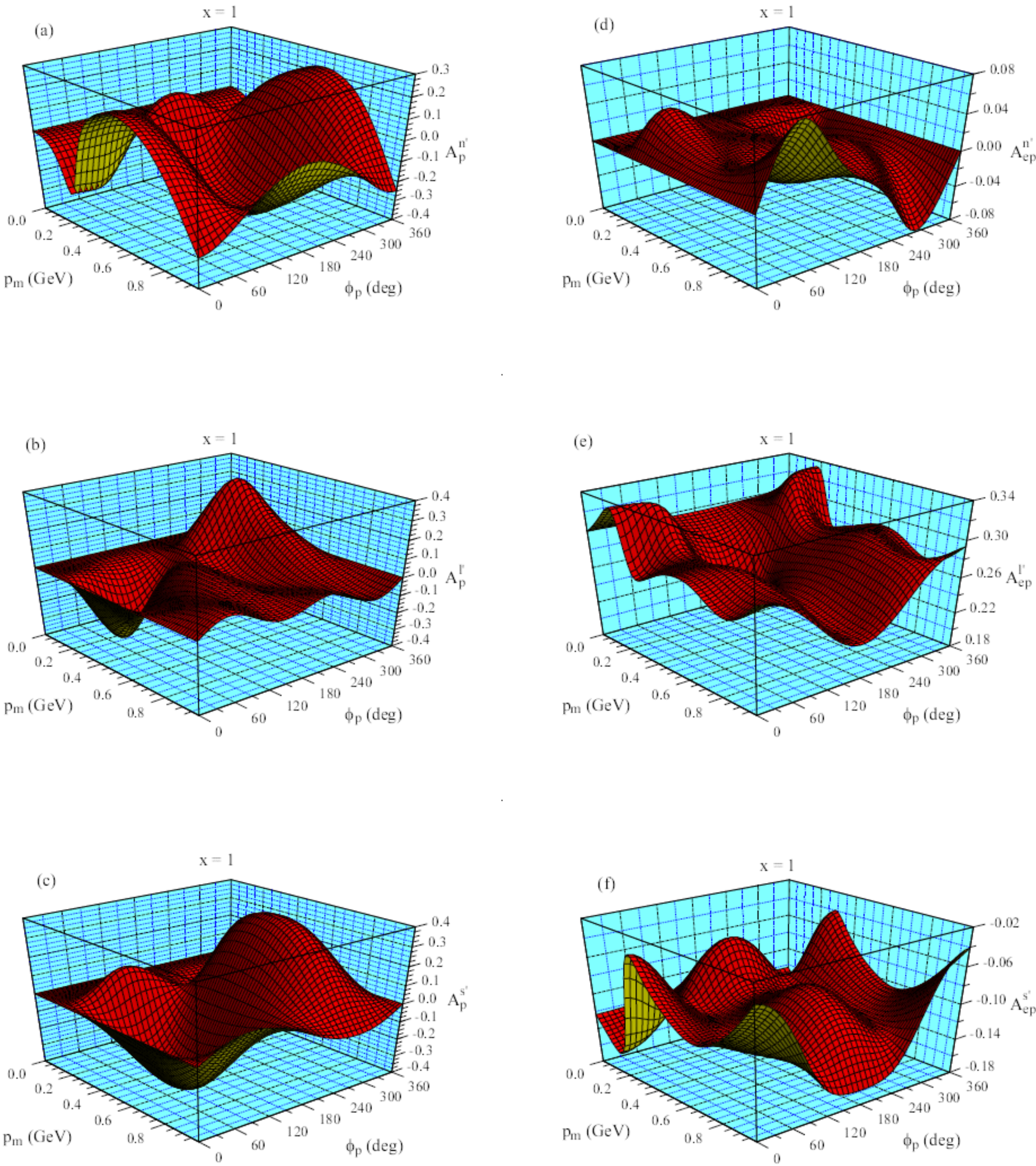
Using $\hat{\bar{\mathcal{S}}} = \hat{n}'$ or $\hat{\bar{\mathcal{S}}} = \hat{l}'$ or $\hat{\bar{\mathcal{S}}} = \hat{s}'$, find:

$\sigma(n') + h\sigma_h(n')$ $\sigma(l') + h\sigma_h(l')$ $\sigma(s') + h\sigma_h(s')$

define the asymmetries:

$$A_p^\xi = \frac{\sigma(\xi)}{\sigma(0)} \qquad A_{ep}^\xi = \frac{\sigma_h(\xi)}{\sigma(0)}$$

$$\xi = n', l', s'$$



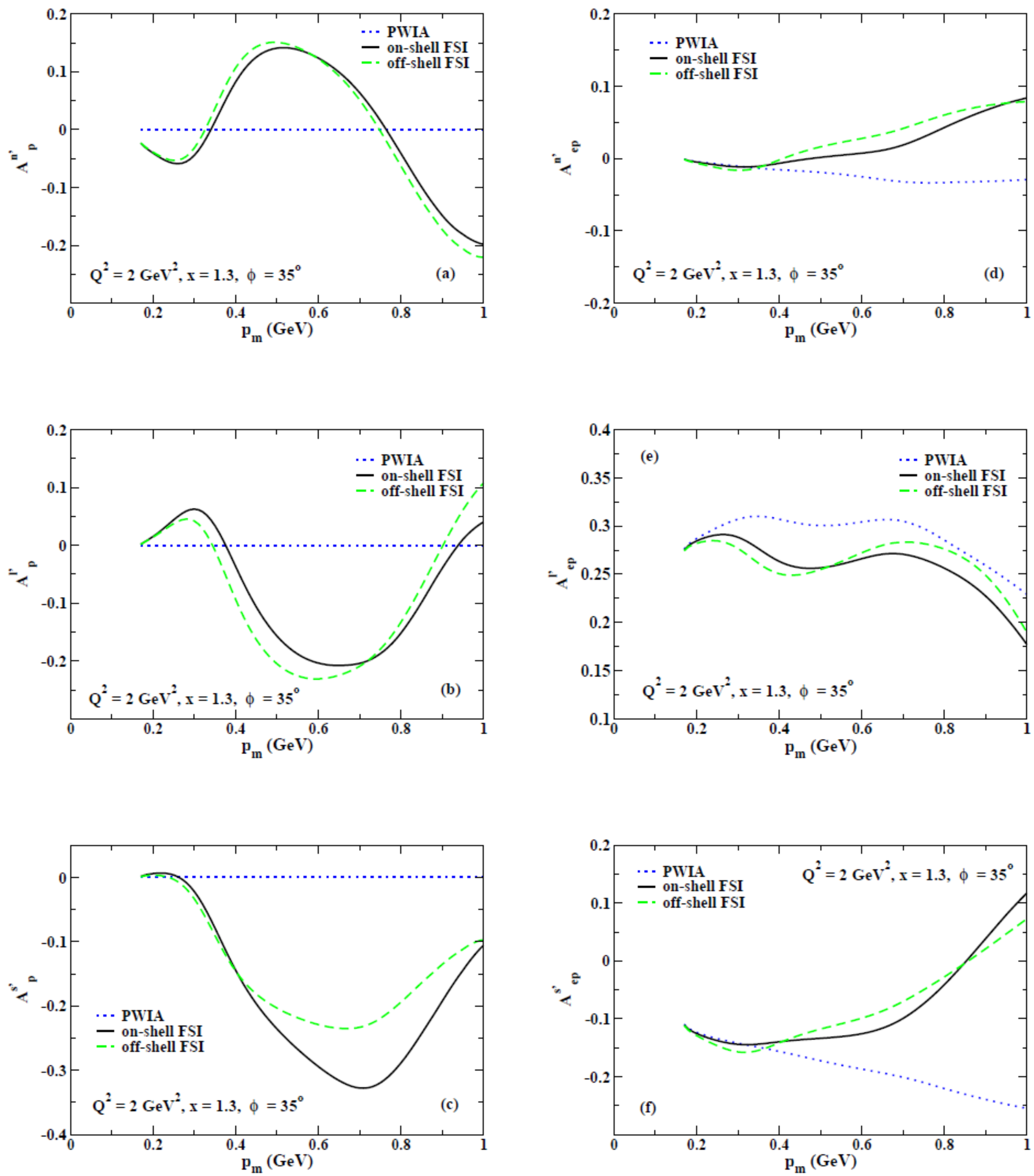
left column:
unpolarized beam

right column:
polarized beam

top row: **normal**
(induced polarization)

middle row:
longitudinal

bottom row:
sideways



Momentum Distributions

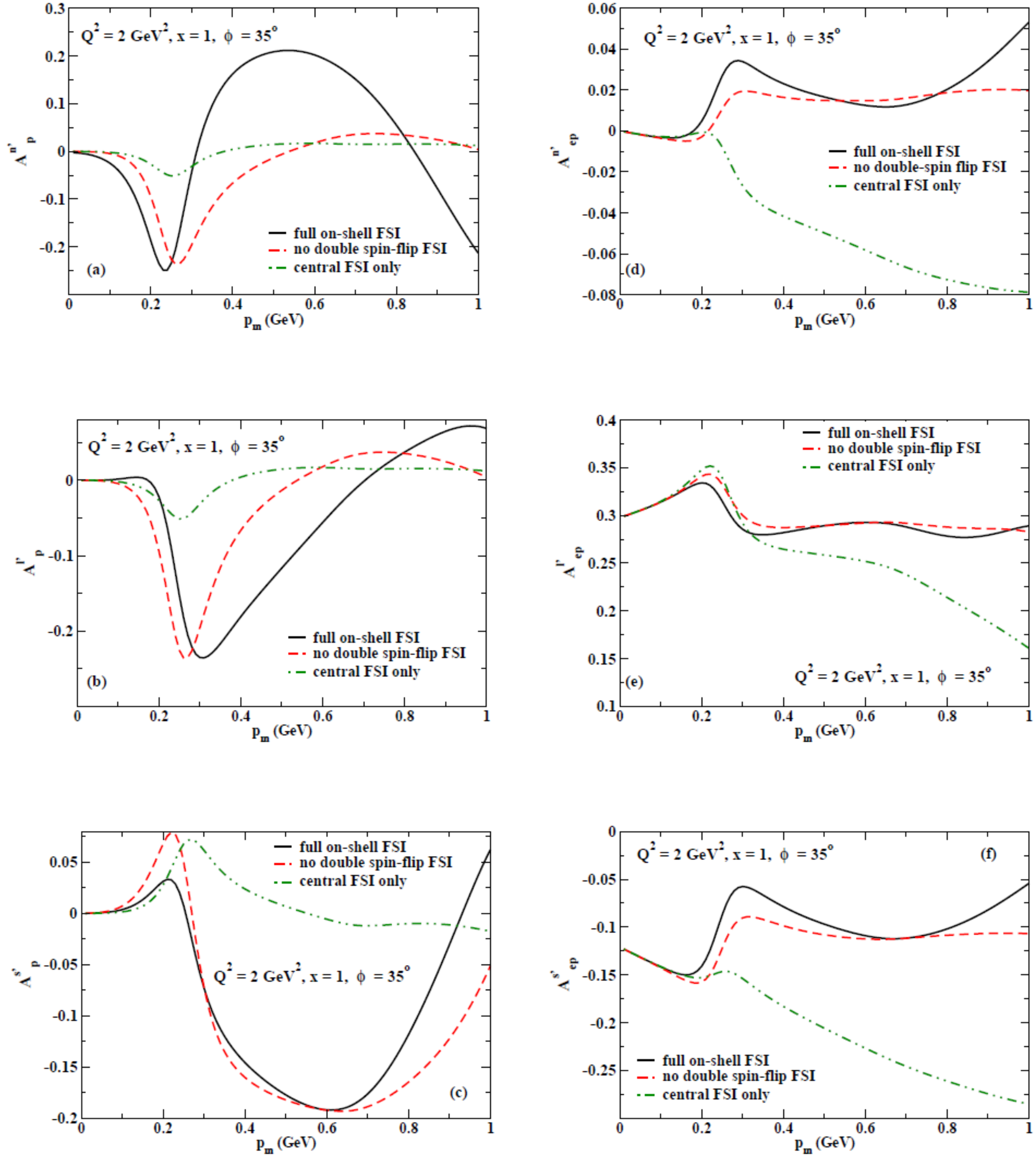
PWIA

on-shell FSI

off-shell FSI

$x = 1$, $Q^2 = 2\ GeV^2$

$\Phi = 35°$



Momentum Distributions

full on-shell FSI

central & spin-orbit FSI

central FSI only

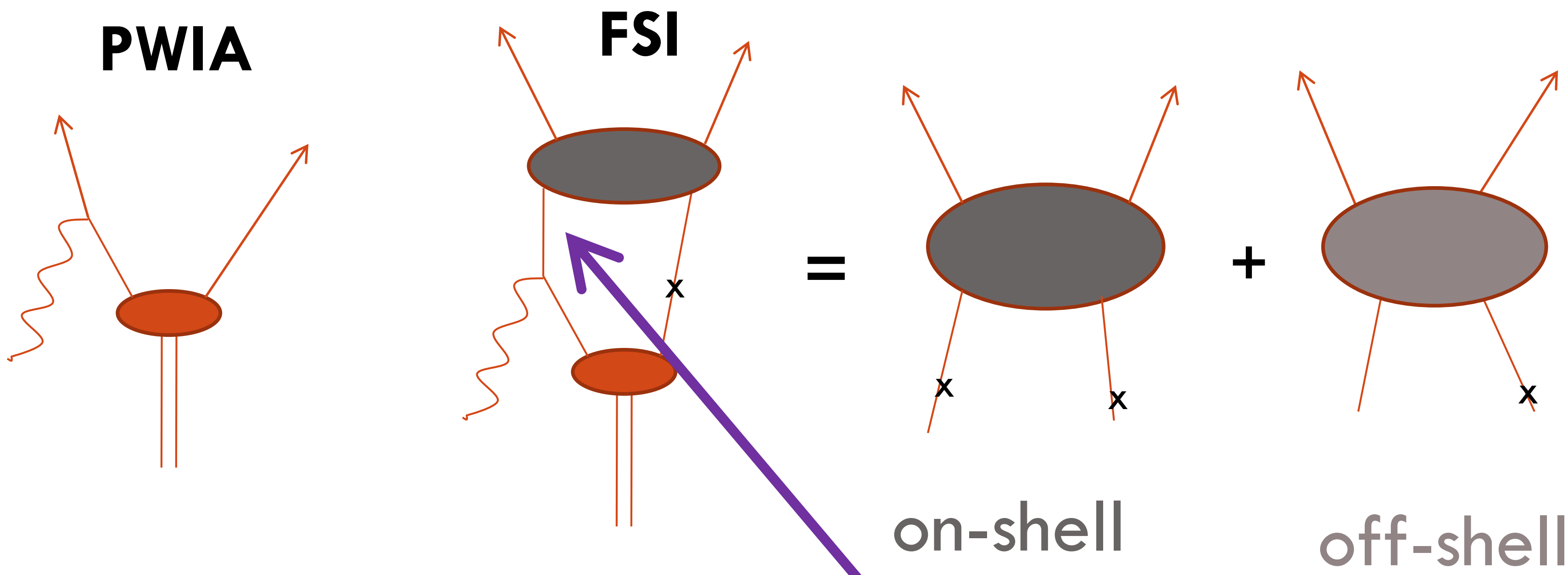
$x = 1,\ Q^2 = 2\ \mathrm{GeV}^2$

$\Phi = 35°$

Polarized Ejected Proton Summary

- FSIs are relevant, at all x
- spin-dependence is important
- need data…

Start of Extra Transparencies



negative energy contribution:
kinematically suppressed, not known, neglected

off-shell FSI, **positive energy contribution**:
requires a dynamical model of the amplitude;
we estimate it with a simple prescription

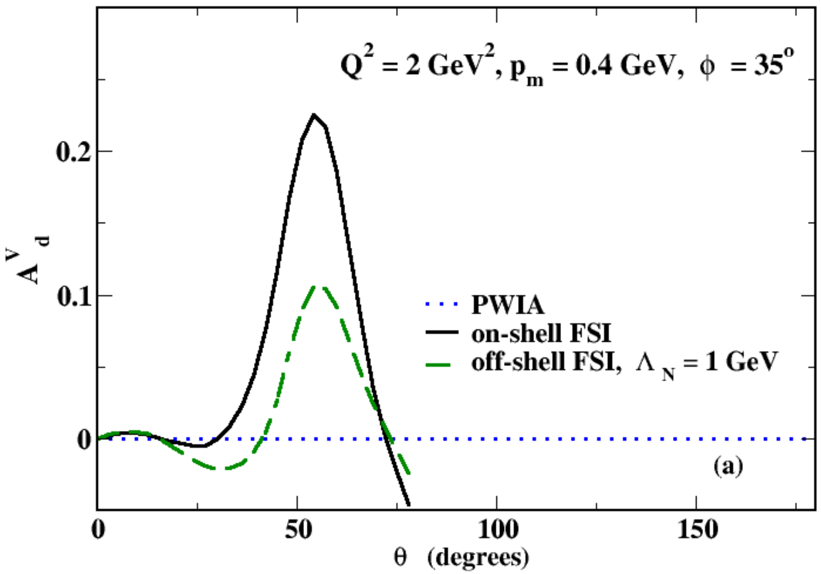
$$\frac{\not p + m}{p^2 - m^2 + i\varepsilon} =$$

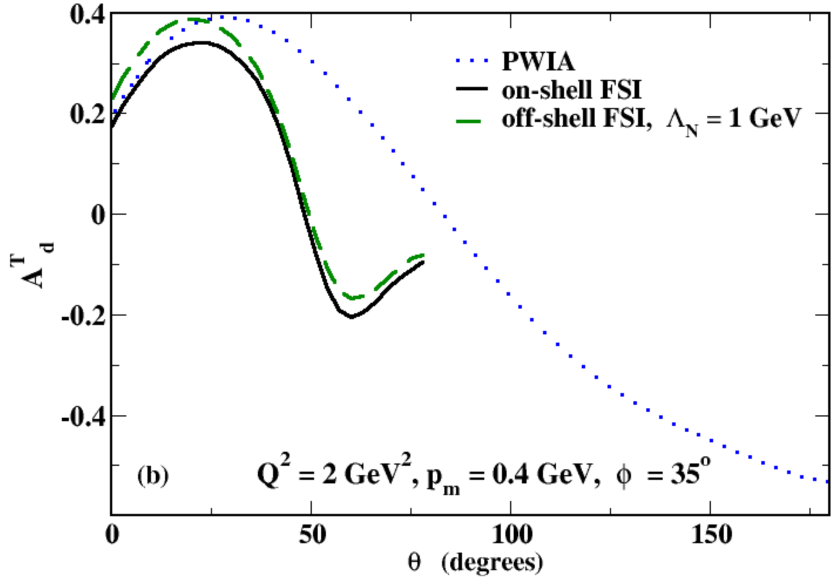
$$\frac{m}{E_p}\sum_s\left(\frac{u(\vec p,s)\bar u(\vec p,s)}{p^0-E_p+i\varepsilon}+\frac{v(-\vec p,s)\bar v(-\vec p,s)}{p^0+E_p-i\varepsilon}\right)$$

FSI matrix element:

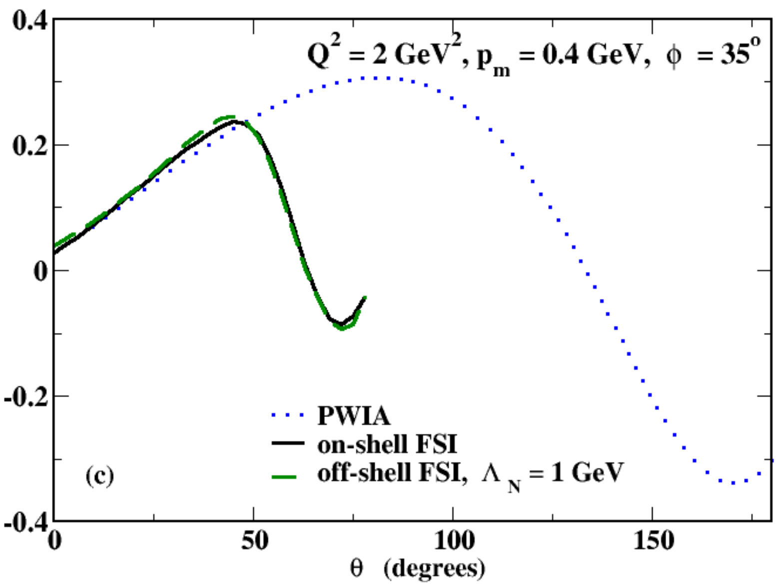
$$\langle \boldsymbol{p}_1 s_1; \boldsymbol{p}_2 s_2 | \, J^\mu_{FSI} \, | \boldsymbol{P} \lambda_d \rangle = \int \frac{d^3 k_2}{(2\pi)^3} \frac{m}{E_{k_2}} \bar{u}_a(\boldsymbol{p}_1, s_1) \bar{u}_b(\boldsymbol{p}_2, s_2) M_{ab;cd}(p_1, p_2; k_2)$$
$$\times G_{0ce}(P + q - k_2) \Gamma^\mu_{ef}(q) G_{0fg}(P - k_2)$$
$$\times \Lambda^+_{dh}(\boldsymbol{k}_2) \Gamma^T_{\lambda_d gh}(k_2, P) \,,$$

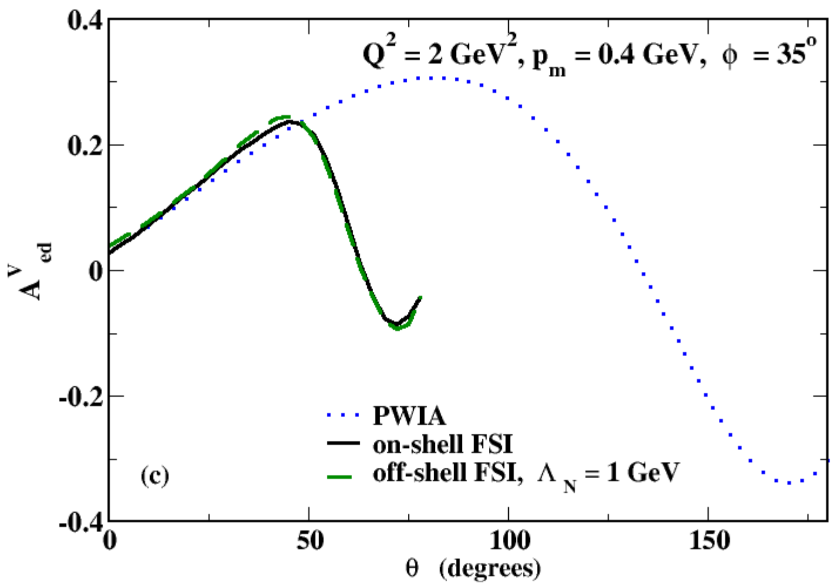
$$\begin{aligned} G_0(p) &= -\frac{m}{E_p} \sum_s \left[\frac{u(\boldsymbol{p}, s) \bar{u}(\boldsymbol{p}, s)}{p^0 - E_p + i\epsilon} + \frac{v(-\boldsymbol{p}, s) \bar{v}(-\boldsymbol{p}, s)}{p^0 + E_p - i\epsilon} \right] \\ &= -\frac{m}{E_p} \left[\frac{\Lambda^+(\boldsymbol{p})}{p^0 - E_p + i\epsilon} - \frac{\Lambda^-(-\boldsymbol{p})}{p^0 + E_p - i\epsilon} \right] \end{aligned}$$

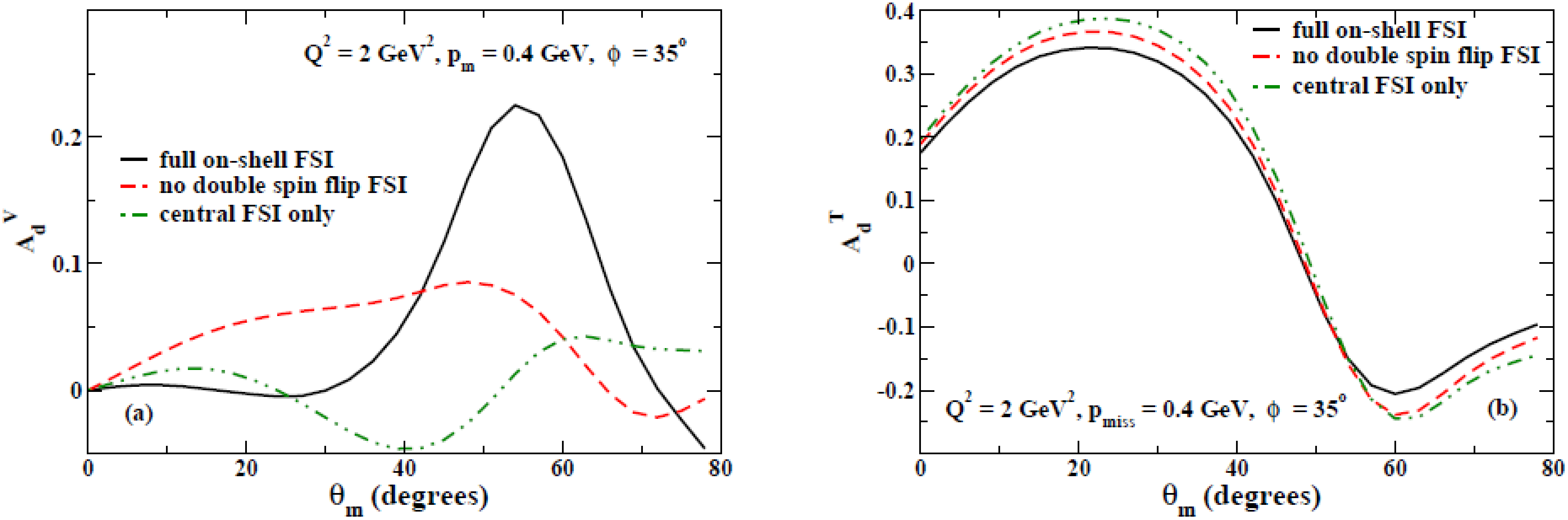




Angular Distributions $p_m = 0.4$ GeV, $Q^2 = 2$ GeV^2







Role of Spin-Dependent FSIs: Single Spin Flip and Double Spin Flip